Notes on Dry Friction

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CASE I

\[ \begin{align*}
\text{Conditions Required For Slipping:} & \quad \frac{A}{\mu_s N} \quad d > 0 \\
\text{Conditions Required For Tipping:} & \quad \frac{A}{\mu_s N} \quad d < 0 \\
\end{align*} \]

**Equilibrium From \( FB \):**

1. \( \sum F_x = 0 \)
2. \( \sum F_y = 0 \)
3. \( \sum M = 0 \)

**To Check For Slipping:**

1. \( \text{Reorient Eqn. With } \sum M \quad d > 0 \)
2. \( \mu_s N - P = 0 \)
3. \( N - W = 0 \)
4. \( P_s + N d - W \left( \frac{d}{2} \right) = 0 \)

**To Check For Tipping:**

1. \( \text{Reorient Eqn. With } \sum F_y \quad d = 0 \)
2. \( A - P = 0 \)
3. \( N - W = 0 \)
4. \( P_s - W \left( \frac{d}{2} \right) = 0 \)

**Validity:**

- \( d \neq \frac{a}{2} \)
- \( \frac{A}{\mu_s N} \neq 0 \)
Friction

Case 2

\[ \frac{a}{b} \]

\[ \frac{c}{d} \]

\[ \frac{e}{f} \]

\[ \frac{g}{h} \]

Two points of contact result in two normal forces, \( N_A + N_B \), values vary as result of loading.

Total frictional force, \( \alpha + \beta \), opposes applied load.

Conditions for tipping...
\[ N_B = 0, \quad \alpha < \frac{m_A}{m_B} N_A \]

Conditions for slippage...
\[ N_A = 0, \quad N_B > 0, \quad N_A > N_B, \quad \beta = \frac{m_B}{m_A} N_A \]

Equilibrium from FBD

\[ \sum F_y = \frac{\alpha}{A} - \frac{\beta}{B} - P = 0 \]

\[ \sum F_x = N_A + N_B - W = 0 \]

\[ \sum M_A = P x + N_B c - W (\frac{g}{2}) = 0 \]

To check for slippage...

Re-write \( \sum M_B \) with \( \frac{\alpha}{A} = \frac{m_B}{m_A} N_A \)

\[ m_A N_A + m_B N_B - P = 0 \]

\[ N_A + N_B - W = 0 \]

\[ P c + N_B c - W (\frac{g}{2}) = 0 \]

Solve for unknowns

Largest valid if
\[ N_A > 0 \text{ and } N_B > 0 \]

To check for tipping...

Re-write \( \sum M_A \) with \( N_A = 0, \alpha + \beta = 0 \)

\[ \alpha - P = 0 \]

\[ N_A - W = 0 \]

\[ P c - W (\frac{g}{2}) = 0 \]

Solve for unknowns

Smallest valid if
\[ \frac{\alpha}{A} \neq \frac{m_B}{m_A} N_A \]
CASE 2

Locating at (L/2, h/2)

Contact surface at angle to horizontal

Object may be acted on by a force

Eliminating all case without external load, P

Equations of motion are:

\[ \begin{align*}
\sum F_x &= \sum F_y = 0 - W\sin \theta = 0 \\
N &= W\cos \theta = 0 \\
M &= W\sin \theta \left( \frac{h}{2} \right) - W\cos \theta \left( \frac{l}{2} \right)
\end{align*} \]

+ \, N(a) = 0

To check for tipping...

Release load, \( F = \frac{W}{2}N \)

\[ \begin{align*}
M_N &= W\sin \theta = 0 \\
N &= W\cos \theta = 0 \\
W\sin \theta \left( \frac{h}{2} \right) - W\cos \theta \left( \frac{l}{2} \right) &= 0
\end{align*} \]

Solve for unknowns. Solution valid if \( 0 < a < \frac{L}{2} \)
**CASE 4**

Object with two points of contact, one smooth (μs > 0) and one rough (μr > 0).

**Impedance Method** - Slippage

\[ f_A = \mu_r N_A \]

\[ + \vec{F}_x = \frac{f_A}{N_A} - N_B = 0 \]

\[ + \vec{F}_y = N_A - W = 0 \]

\[ \vec{M}_A = N_B (L \sin \theta) - W \left( \frac{L}{2} \cos \theta \right) = 0 \]

Conditions for case where \( f_A = \mu_r N_A \)

\[ M_s N_A - N_B = 0 \]

\[ N_A - W = 0 \]

\[ N_B \sin \theta - W \left( \frac{L}{2} \cos \theta \right) = 0 \]

Simplifying...

\[ N_A = M_s N_A \]

\[ N_A = W \]

\[ M_s \sin \theta - W \left( \frac{L}{2} \cos \theta \right) = 0 \]

\[ M_s \sin \theta = \frac{L}{2} \cos \theta \]
**CIRCULAR OBJECT WITH APPLIED FORCE (OR MOMENT)**

Friction present at both points of contact A and B.

**FBD**

Assumed direction of impending motion is clockwise, i.e., up and to the right, which has directions that oppose this motion.

**Equilibrium**

\[ N_A - \frac{F_B}{R} = 0 \]

\[ \sum \mathbf{F} = 0 \]

\[ \sum \mathbf{M}_A = 0 \]

For impending slippage...

\[ N_A > 0 \] and \[ N_B > 0 \] and \[ \frac{F_A}{m} = m \cdot N_A \] and \[ \frac{F_B}{m} = m \cdot N_B \]
\[ P \rightarrow \frac{L}{2} = \frac{m}{2} N_B \]

\[ F_x = P + \mu_N N_B - N_A = 0 \]
\[ F_y = N_B - W = 0 \]
\[ M_B = N_A (L) - w (2) = 0 \]

**Solving...**

\[ N_B = W \quad N_A = \frac{2}{3} W \]
\[ P = \frac{2}{3} W - \mu_s W = W \left[ \frac{2}{3} - \mu_s \right] \]
\[ P = (0.60 \times 10^3)(1.8 \times 10^2) \left[ \frac{2}{3} - 0.2 \right] \]
\[ P = 122,760 \] **Note:** To prevent rod from sliding down.

**FBD**

\[ N_B \]
\[ W \]
\[ P \]
\[ N_A \]
\[ N_B = \frac{2}{3} W \]

\[ P = (0.60 \times 10^3)(1.8 \times 10^2) \left( \frac{2}{3} - 0.2 \right) \]
\[ P = 255,500 \] **Note:** To cause rod to slide up wall.

**Problem 2**

**FBD A**

**FBD B**
PROBLEM 2

Proper force have directions that oppose collinear motion due to P.

Forces: \( F_A, F_B, P, f_A, f_B \)

Conditions: \( N_A, N_B, P, \theta_A, \theta_B \)

\[ \text{Dimensions not given so it is possible that } X_A \text{ and } X_B \text{ are not needed} \]

\[ -X_A + X_B + D = 0 \quad (1) \]

\[ + \quad (X_B - T + P \cos 30° = D) \quad (2) \]

\[ -W + N_A = 0 \quad (3) \]

\[ + \quad (N_B - W + P \sin 30° = D) \quad (4) \]

For both to move: \( X_A = \mu_s N_A \) and \( X_B = \mu_s N_B \)

\( N_A, N_B, T, P \) \( \Sigma \) 4 unknowns can be solved for using 4 equations...

\[ \text{For the analysis, the force systems on each body can be treated as concurrent.} \]

\[ \begin{align*}
\text{Find A:} & \quad N_A, F_{BA} \\
\text{Free Body Diagram:} & \\
N_A & \quad \downarrow \\
W & \quad \uparrow \\
T & \quad \rightarrow \\
F_{BA} & \quad \rightarrow \\
\text{Solving for } P: & \\
(1) & \quad T = X_A = \mu_s N_A \\
(2) & \quad N_A = W \\
(3) & \quad N_B = W - P \sin 30° \\
(4) & \quad -M_a [W - P \sin 30°] - M_a W + P \cos 30° = D \\
& \quad -M_a W + M_a P \sin 30° - M_a W + P \cos 30° = D \\
& \quad P [M_a \sin 30° + \cos 30°] = 2M_a W \\
& \quad P = \frac{2M_a W}{M_a \sin 30° + \cos 30°} = \frac{2(10,000)(30°)}{0.25 \sin 30° + \cos 30°} \\
& \quad P = 247 \text{ N} \\
\end{align*} \]
\[ \begin{align*}
+ \Delta F_x &= N_a - P_b = D \\
+ \Delta F_y &= L_a + N_b + P - W = D \\
\Delta M_o &= P c - a r - N_a r = D
\end{align*} \]

Imposing moment:
\[ \Delta M_a = M_s N_a - M_s N_b = 0 \]

Substituting:
\[ \begin{align*}
(1) & \quad N_a = M_s N_b \\
(2) & \quad M_s N_a + N_b + P - W = D \\
(3) & \quad P c - M_s N_a r - M_s N_b r = D \\
(4) & \quad N_b = \frac{w - P}{M_s^2 + 1} \\
(5) & \quad P c = N_b \left[ \frac{M_s^2 r + M_s r}{M_s^2 + 1} \right] = \frac{w - P}{M_s^2 + 1} \left( \frac{M_s^2 r + M_s r}{M_s^2 + 1} \right)
\end{align*} \]

\[ P [0.6] = \left[ (100)(0.3) - P \right] \left[ (0.322)(0.9) - (0.322)(0.9) \right] \]

\[ 0.6 P = 0.322 \left[ 981 - P \right] = (0.322)(981) - 0.322 P \]

\[ P [0.922] = (0.322)(981) \]

\[ P = 343 N \]

**Problem 4**

\[ \begin{align*}
\text{Eqs. of static equilibrium} \\
+ \Delta F_x &= P - A = D \\
+ \Delta F_y &= N - W = D \\
\Delta M_a &= W (4.5) - N d - P (4.5) = D
\end{align*} \]
For tipoff, $d > D$  

$P = \frac{L}{4}$  

$N = W$  

$W = 4.5P$  

$P = \frac{1.5}{4.5}W = \frac{1.5}{3} = 0.5W = 83 \text{ lb}$  

$N = 25p \text{ lb}$  

$q = 83 \text{ lb}$  

$q_{\text{max}} = M_{x}N = (0.4)(250) = 100 \text{ lb}$  

$q < q_{\text{max}}$, so place tips before sliding can occur.  

$\therefore P = 83 \text{ lb}$