INTRODUCTION

Imagine a dipole antenna composed of a thin perfectly conducting wire of length $2h$ and placed along the $z$ axis, as shown in Figure 1. A small gap exists at its center. A time harmonic, linearly polarized, plane electromagnetic wave is incident from an arbitrary direction. The polarization direction of the accompanying electric field is also arbitrary. Our problem is to find the complex phasor voltage, $V$, in the gap. This would appear to be one of the most fundamental problems in the theory of the dipole antenna. However, a student who has completed an elementary course in antenna theory that sticks closely to any of three popular textbooks in this subject [1] [2] [3] might find that she or he has difficulty with this receiving dipole. This person could also find it challenging to obtain the voltage created across an arbitrary load placed in the gap, or might find it hard to solve the problem of determining the voltage created when the antenna is in proximity to a conducting ground plane.

Figure 1. A simple receiving dipole.
The incident electric field is created by a loop of diameter 2a.

**Figure 2b.** A voltage is induced in a gap in the loop.
In treating the problem of signal reception by an antenna, most textbooks and handbooks understandably give emphasis to the concept of the antenna's effective area (or aperture), \( A \) (e.g., [2, Chapter 2]). There is much to recommend the use of effective area: it connects the strength of the incident flux of power at the antenna to the power appearing in an ideal load placed at the antenna's terminals. Moreover, effective area is directly related to antenna gain, \( G \), through the well-known formula 
\[
G = \frac{4\pi A}{\lambda^2}
\]
‘and effective area appears explicitly in the Friis transmission formula relating transmitted power from one antenna to received power at another [1], [p.37].

However, there are situations where the use of antenna effective length, especially vector effective length, has an advantage over effective area. Effective area is of no use in solving the problem of obtaining the complex phasor voltage in the gap in the antenna just cited. Effective area by itself contains no vector information, and does not account for any loss in power to the load due to polarization mismatch between the receiving antenna and the polarization of the incident wave. In contrast to effective area, effective length allows us to determine the complex phasor voltage in the Thevenin equivalent circuit of a receiving antenna. Finally, the front-end sensitivity of many receivers is stated in microvolts, and it is thus advantageous to know how many volts a receiving antenna will supply to a receiver. Effective length yields this number, while the use of effective area requires an additional calculation.
In what follows, we treat this subject in such a way that anyone who is familiar with the principle of reciprocity, Faraday’s law, Thevenin’s theorem, and who has been taught in an antennas course how to find the far-zone electric field of a very small loop antenna, can deal with the preceding questions. We proceed by using the concept of antenna effective length.

A Classroom Lecture

2.1 Scalar Effective Length

We begin with the simplest possible example, and study a thin-wire receiving dipole of length $2h$, placed along the $z$ axis, with a gap at its center. A linearly polarized plane wave is incident, as shown in Figure 2a, where for the moment we assume that the direction of polarization is such that the electric field is in the plane determined by the antenna and the direction of propagation of the wave. The incident electric field, $E_0^i a_\theta$, is in the same vector direction as that of the electric field that would have been produced if we had been transmitting with this antenna: i.e., transmitting with an upward-directed phasor current, $I$, entering the upper half of the dipole at the center, which means that far zone electric field radiation is in the direction of the unit vector $a_\theta$. Let us assume that this incident field arises from a specific source—a distant loop antenna—shown in Figure 2a. The centers of the loop and the dipole are separated by a distance $r$. The field of an electrically small circular loop antenna is derived in most courses in antenna theory (e.g., [2, Section 7.5]). If the loop is of radius $a$, and if it carries a current $I_0$ circulating in the
counterclockwise direction as shown, the electric field in the loop's plane at \( r \) meters from the loop's center is
\[
E = \eta \frac{k^2 a^2 I_0}{4r} e^{-jkr},
\]
(1)
where \( \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \) and \( k = \frac{2\pi}{\lambda} \).

Here, the direction of the field is in the same sense as the current flow. We want this to be identical to the given field striking the antenna. Hence,
\[
E_0^i = \eta \frac{k^2 a^2 I_0}{4r} e^{-jkr},
\]
(2)
which we solve to obtain
\[
I_0 = \frac{4r}{\eta k^2 a^2} E_0^i e^{jkr}.
\]
(3)
This is the current on the loop that will create the given electric field striking the center of the dipole.

Now, we remove the current generator supplying the loop antenna with current, and leave behind a small gap. As shown in Figure 2b, we fill the center of the dipole with a current generator supplying the current \( I = I_0 \) found in Equation (3).

We next apply the theory of reciprocity, familiar to most students from their courses in circuit theory. If they need convincing that the formula applies to circuits that are coupled by electromagnetic radiation, they may refer to [5]. The voltage in the gap of the loop, \( V_{1,2} \), must be identical to that which had appeared in the gap of the dipole when the loop was supplied with current \( I_0 \). This voltage is easily found using Faraday's law.
of induction. For the moment, we assume that the dipole is electrically very thin, and that \( kh \neq n\pi \), where \( n \) is any integer. Most textbooks on antennas or electromagnetic theory assume that the upward current, \( I_z(z) \), along the thin center-driven dipole (if it is less than one or two wavelengths in size) is then well approximated by

\[
I_z(z) = \frac{I_0 \sin(kh - k|z|)}{\sin(kh)}.
\]

Students learn (see, e.g., [6]) that the far-zone fields created by a center-driven dipole with this current distribution are given by

\[
E_\theta = j\eta \frac{I_0}{2\pi r \sin(kh)} F(\theta) e^{-jkr} \tag{5}
\]

and

\[
B_\phi = j \frac{\mu_0 I_0}{2\pi r \sin(kh)} F(\theta) e^{-jkr}, \tag{6}
\]

where \( \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \). Here,

\[
F(\theta) = \frac{\cos(kh \cos \theta) - \cos(kh)}{\sin \theta} \tag{7}
\]

The loop is assumed to be small enough so that the magnetic field striking its center is identical to the magnetic field striking anywhere in its enclosed area. The magnetic field caused by whatever current appears on the open-circuited loop due to incident fields is assumed negligible. In Figure 2b, the magnetic
flux, $\Phi$, threading downward through the plane of this page when the dipole is driven with current $I_0$ is thus the product of the magnetic-flux density in Equation (6) with the area of the loop:

$$\Phi = j \frac{\pi a^2 \mu_0 I_0}{2\pi r \sin (kh)} F(\theta) e^{-jkr}. \quad (8)$$

By Faraday's law of induction (i.e., here, $V_{12} = -j\omega \Phi$), this implies that the voltage in the gap is

$$V_{12} = \frac{\omega \pi a^2 \mu_0 I_0}{2\pi r \sin (kh)} F(\theta) e^{-jkr}. \quad (9)$$

Using $I_0$ from Equation (3) in the preceding, we have

$$V_{12} = \frac{\lambda}{\pi \sin (kh)} F(\theta) E_0^i. \quad (10)$$

where we have used $\omega \mu_0 = k\eta$.

We know by reciprocity that $V_{12}$ in Equation (10) is the voltage induced in the gap in the dipole in Figure 2a.

We now make this definition:

*If a receiving antenna is exposed to a linearly polarized electric field associated with a plane wave coming from a particular direction, and if this field has the identical (linear) polarization to that which would occur if the antenna were transmitting in that direction, then the product of the (scalar) effective length of the antenna, $l_{eff}$, for that direction and the incident phasor*
electric field will yield the voltage induced in a small gap in this antenna. The incident field is defined at the gap.

The preceding tacitly assumes that we are dealing with an antenna that when transmitting produces a linearly polarized wave in its far zone. There is also an implicit sign convention: If the voltage appearing in the gap when we are receiving is designated $V_{12}$, then the electric field created when transmit is such that the current enters terminal 1.

With this definition, we see that the effective length of the dipole studied here is the ratio of voltage to field from Equation (10):

$$l_{eff} (\theta) = \frac{\lambda}{\pi \sin (kh)} F(\theta) = \frac{V_{12}}{E_0^i}.$$  \hspace{1cm} (11)

Notice how the transmitting pattern of the dipole, $F(\theta)$, establishes the receiving property $l_{eff} (\theta)$. The preceding can yield some results that might look familiar. If we are employing a half-wave dipole, where $k_h = \pi / 2$, then Equations (7) and (11) yield

$$l_{eff} (\theta) = \frac{\lambda \cos \left[ \left( \frac{\pi}{2} \right) \cos \theta \right]}{\pi \sin \theta} \hspace{1cm} (12)$$

from which, if $\theta = 90^\circ$, Equation (12) tells us that the effective length is $\frac{2}{\pi}$ times the actual antenna's length.
If when driven at the center the dipole is electrically short, so that \( kh \ll 1 \), the distribution of current along its length is approximately triangular. Using small-argument expansions
\[
\sin(kh) = kh, \quad \cos(kh) = 1 - \frac{1}{2}(kh)^2,
\]
and
\[
\cos(kh \cos \theta) = 1 - \frac{1}{2}(kh)^2 \cos^2 \theta
\]
in Equations (7) and (11), we have for a short antenna,
\[
\ell_{\text{eff}}(\theta) = h \sin \theta .
\]

The effective length of an electrically short receiving antenna is thus at best \( h \), half the actual length \( 2h \), and this occurs when \( \theta = \frac{\pi}{2} \).

**2.2 Vector Effective Length**

Suppose the antenna in Figure 2a had been exposed to a plane wave the electric field of which possessed a component \( E_{on}^i a_\phi \), as shown in Figure 3a. This component is normal to both the direction of the electric field produced in the far zone by the antenna when transmitting, and to the direction of propagation of that wave. We can use reciprocity and the loop antenna to argue that this new component, \( E_{on}^i a_\phi \), will have no effect on the open-circuit voltage appearing in the gap. We simply imagine this component to have been produced by a current-carrying loop antenna with a normal in the direction \( a_\theta \) as shown in Figure 3 a.

Now, this same current, when supplied to the dipole, will produce a far-zone magnetic field lying only in the direction \( a_\phi \). No magnetic flux lines pass through the plane of the loop (see Figure 3b), and Faraday’s law indicates there is no voltage induced in the loop. By reciprocity, no voltage will thus appear in
the gap of the dipole when exposed to the electric field shown in Figure 3a. The dipole responds only to the component of electric field in the direction of $\mathbf{a}_\theta$. The preceding can be generalized:

*A receiving antenna, with an open circuit (a gap) for its load, exposed to an incoming plane electromagnetic wave of arbitrary linear polarization, will produce a voltage in the gap in response to only that portion of the wave the electric field of which is along the direction of the electric field created by the antenna if it were transmitting in the direction from which the incoming radiation is incident.*

Since it speaks of the direction of the electric field, the above assumes that the antenna creates a linearly polarized wave when transmitting into its far zone.

From the above statement, it becomes natural to define a *vector effective length* for a receiving antenna: The vector effective length of a receiving antenna, $l_{\text{eff}}(\theta, \phi)$, exposed to a plane wave arriving from direction $\theta, \phi$, is the product $l_{\text{eff}}(\theta, \phi)\mathbf{u}(\theta, \phi) = l_{\text{eff}}(\theta, \phi)$, where $l_{\text{eff}}(\theta, \phi)$ is the (scalar) effective length for reception from that direction, and $\mathbf{u}(\theta, \phi)$ is the unit vector in the direction of the electric field created in the direction $\theta, \phi$, if we use the receiving antenna for transmitting. The voltage induced in the gap of the antenna by an incident electric field is thus given by

$$V_{1,2} = l_{\text{eff}}(\theta, \phi) \cdot \mathbf{E}_0(\theta, \phi).$$

(14)
The use of the vector effective length and Equation (14) thus ensure that the voltage induced in the gap of a receiving antenna is a response only to that portion of the incident electric field having the polarization that causes the voltage to appear.

2.3 Further Generalizations and More-Complicated Antennas

The electric field in the far zone of an antenna radiating into free space can be cast into the form

$$\mathbf{E} = j\eta \frac{kI_0}{4\pi r} \left[ m(\theta, \phi) \mathbf{a}_\theta + n(\theta, \phi) \mathbf{a}_\phi \right] e^{-jkr}, \quad (15)$$

where $I_0$ is the current supplied by the generator feeding the antenna. We assume this generator to be located at the origin. Justification for this equation can be found in [6, Section 12.4], and would be taught in most courses in antenna theory under the rubric “far-field approximation.” We have modified the traditional expression by including a factor $I_0$ for the phasor current supplied by a (single) source of energy. With an application of $\nabla \times \mathbf{E} = -j\omega \mathbf{B}$ to the above, and retention of only the far-zone terms, we have that the magnetic flux density is

$$\mathbf{B} = j\mu \frac{kI_0}{4\pi r} \left[ -n(\theta, \phi) \mathbf{a}_\theta + m(\theta, \phi) \mathbf{a}_\phi \right] e^{-jkr}. \quad (16)$$
Figure 3a. The incident electric field is normal to the wire dipole.

Figure 3b. The magnetic flux density vector is parallel to the plane of the loop.
If this antenna is used for receiving, with incident fields at the origin having components $E_{\theta 0}^i a_\theta$ and $E_{\phi 0}^i a_\phi$, we can obtain the voltage produced in the gap by following the method of reciprocity, as used above. We will require the use of two small loops to produce the two components of field. If only $E_{\theta 0}^i a_\theta$ is incident, then we assume it to have been produced by a loop antenna such as that shown in Figure 2a, where the current is given in Equation (3), and we take $E_{\theta 0}^i$ as $E_{\theta 0}^i$. Applying this same current to the dipole, determining the voltage, $V_{12}$, by means of Faraday’s law, and noting that only the portion of the magnetic flux density $j \mu \frac{k I_0}{4 \pi r} \left[ m(\theta, \phi) a_\theta \right] e^{-jk r}$ is required for our calculation, we proceed as above and find $V_{12} = m(\theta, \phi) E_{\theta 0}^i$.

If just $E_{\phi 0}^i a_\phi$ was incident, we would assume it to have been created by a loop antenna such as that shown in Figure 4a. The current circulating in the direction of the arrow on the loop would have to be

$$I_0 = -\frac{4 r}{\eta k^2 a^2} E_{\phi 0}^i e^{jk r}. \quad (17)$$

Note the minus sign. If we apply this current to the antenna and compute the voltage induced in the loop of Figure 4b by means of Faraday’s law and the vector component of the magnetic flux density $j \mu \frac{k I_0}{4 \pi r} \left[ -n(\theta, \phi) a_\theta \right] e^{-jk r}$, we find that the voltage, $V_{12}$, is $n(\theta, \phi) E_{\phi 0}^i$. With electric field $E_{\theta 0}^i a_\theta + E_{\phi 0}^i a_\phi$, incident on the receiving antenna and striking a gap in the antenna at the origin of the coordinate system, we thus have a voltage across the gap
From this we can make the following generalization: If the far-zone electric field of an antenna in free space, with generator at the origin supplying current \( I_0 \), is stated in the form:

\[
V_{12} = m(\theta, \phi)E^{i}_{\theta 0} + n(\theta, \phi)E^{i}_{\phi 0}
\]

\[
= \left[ m(\theta, \phi)a_\theta + n(\theta, \phi)a_\phi \right] \cdot \left( E^{i}_{\theta 0}a_\theta + E^{i}_{\phi 0}a_\phi \right)
\]

\[
E = j\eta \frac{kI_0}{4\pi r} \left[ m(\theta, \phi)a_\theta + n(\theta, \phi)a_\phi \right] e^{-jk\phi} \quad (18)
\]

**Figure 4a.** The component \( E^{i}_{\phi 0} \) is created by a loop with the plane normal to \( a_\theta \).
and if the polarization of the field is linear, then the vector effective length of the antenna is

\[ l_{\text{eff}}(\theta, \phi) = m(\theta, \phi) a_\theta + n(\theta, \phi) a_\phi. \]  \hspace{1cm} (19)

The voltage induced in the gap of the antenna by an incident linearly polarized plane wave is given by Equation (14).

The assumption of linear polarization means that \( m(\theta, \phi) \) and \( n(\theta, \phi) \) are in phase or out of phase. We can immediately put the preceding to work. It is a standard exercise (see [3, p. 159]) to show that an electrically short antenna, also known as the elementary or Hertzian dipole, of length \( 2h \) (or \( 2h = L \)), centered at the origin and having a uniform upwardly directed current, \( I_0 \), along the z axis, creates an electric field

\[ E = j\eta \frac{kI_0}{4\pi r} 2h \sin \theta a_\theta e^{-jkr}. \]

From this, we see that the vector...
effective length is \( l_{\text{eff}}(\theta, \phi) = 2h \sin \theta a_\theta \). This is a well-known result.

One can of course apply Equations (18) and (19) to antennas that are not necessarily confined to the \( z \) axis. A good exercise is to derive the vector effective length for the electrically short 90° Vantenna shown in Figure 5. Here, we assume that that the two legs of the antenna are so short that the current distribution on them, when transmitting, is given by \( I_z = I_0 (1 - z/h) \) and \( I_x = -I_0 (1 - x/h) \) on the wires, except in the small gap containing the current generator. The resultant vector effective length is

\[
l_{\text{eff}}(\theta, \phi) = a_\theta \frac{1}{2} h (\sin \theta + \cos \theta \cos \phi) - a_\phi \frac{1}{2} h \sin \phi.
\]

The preceding indicates that if a plane wave were incident on the antenna, and polarized only in the direction of \( a_\theta \), then the resultant voltage in the gap will be maximum if we choose \( \phi = 0 \) and \( \theta = \pi/4 \) radians, which is intuitively satisfying.

Another possible exercise is to show that for a wire dipole antenna aligned along the \( z \) axis, driven with current \( I_0 \), and having a current \( I_z(z) \) along its length, the far-zone electric field can be given by

\[
E = j \eta \frac{k I_0}{4 \pi r} \left[ m(\theta) a_\theta \right] e^{-jkr},
\]

where
and the integral is taken along the portion of the $z$ axis carrying the wire. The effective length is given by $m(\theta)$, which simplifies to

$$m(\theta) = \frac{1}{I_0} \int I_z(z) e^{jkz\cos\theta} \, dz,$$

$$m(\theta) = \frac{1}{I_0} \int I_z(z) \, dz \quad \text{if} \quad \theta = \pi/2. \quad (20)$$

### 2.4 Antenna Driving a Load

An antenna usually doesn't have an open circuit as its load. Most students are willing to accept that Thevenin's theorem, learned in their courses in circuit theory, will still be valid when the coupling between the input and output of a system is by means of electromagnetic waves. If we thus place a complex impedance in the gap of any of the antennas dealt with so far, the voltage across this load can be computed, if we use the Thevenin equivalent circuit shown in [Figure 6](#). Here, the Thevenin driving voltage is simply the voltage appearing in the gap if the load were replaced by an open circuit. Thus, $V_{th} = E_{inc} \cdot I_{eff}(\theta, \phi)$. The Thevenin impedance is that impedance seen looking into the antenna when all sources of energy in the universe have been removed. For a lossless antenna in free space, this impedance would be of the form $Z_{th} = R_r + jX$ where $R_r$ is the radiation resistance of the antenna, while $X$ is the effective series reactance seen looking into the antenna. More generally, $Z_{th} = R_r + R_\Omega + jX$, where $R_\Omega$ takes into account ohmic losses occurring when the antenna is used for transmitting. Let a conjugate matched load be applied between terminals 1 and 2 of
the receiving antenna, so that \( Z_L = R_r + R_\Omega - jX \). This load now appears between terminals 1 and 2 of the circuit in Figure 6. We obtain the maximum available power at the load, which is

\[
W_L = \frac{V_{th} V_{th}^*}{2 \sqrt{2}} \frac{1}{2 \sqrt{2}} \frac{1}{R_r + R_\Omega}
\]

\[
= \frac{(E_{inc} \cdot I_{eff})(E_{inc} \cdot I_{eff})^*}{8(R_r + R_\Omega)}.
\]

Figure 5. A V antenna.
If the incident electric field, which is linearly polarized, is oriented along the vector effective length, $l_{\text{eff}}$, the preceding expression is maximized and simplifies to

$$W_L = \frac{1}{8} \frac{|E_{\text{inc}}|^2 |l_{\text{eff}} (\theta, \phi)|^2}{R_r + R_\Omega}, \quad (21)$$

where the magnitude signs refer to the magnitude of complex scalar phasors.

For a linearly polarized plane wave, the time-averaged flux of power in watts per square meters is $P = \frac{|E_{\text{inc}}|^2}{2\eta} = \frac{|E_{\text{inc}}|^2}{240\pi}$. With this used in Equation (21), we obtain

$$W_L = 30\pi \frac{P |l_{\text{eff}} (\theta, \phi)|^2}{R_r + R_\Omega} = PA_{\text{eff}} (\theta, \phi), \quad (22)$$

where
is known as the maximum effective area of a receiving antenna. Its use assumes that the incident radiation is polarized so as to maximize the power to the load, and it relates the incident time-averaged Poynting vector to the power delivered to an ideal load. The electric field in this instance must be along $l_{\text{eff}}(\theta, \phi)$.

It is commonly assumed that the radiation resistance of a thin half-wave dipole is about 73.2 ohms. Neglecting any ohmic losses in the antenna, so that $R_\Omega = 0$, and using Equations (12) and (23), we obtain the effective area of a half-wave dipole of length $2h$ as

$$A_{\text{eff}}(\theta, \phi) = 2.08h^2 \frac{\cos^2 \left( \frac{\pi}{2} \cos \theta \right)}{\sin^2 (\theta)}.$$  

The preceding discussion should be sufficient for a first classroom lecture on effective length in an elementary course. However, some instructors may wish to consider how to use the concept of effective length when the receiving antenna is a base-loaded monopole placed over an infinite perfectly conducting ground plane. This matter is treated here in the Appendix.
COMMENTARY

To see the advantages of the argument just presented, let us see how three standard texts treat the receiving antenna. Reference [1, p. 397] said, “The interaction of the incident electric field $E^i$ with the receiving antenna is facilitated by the concept of vector effective length of an antenna, $h$, which is defined through $V_A = E^i \cdot h^*$ where $V_A$ is the open circuit voltage across the antenna terminals.... The receiving antenna relation applies to any antenna and is very intuitive.” The need for taking the conjugate of $h$ in the preceding was never explained, and indeed the conjugate does not appear in handbook formulas on this subject, e. g., [4]. Reference [1] gave as an example the computation of the vector effective length of an “ideal dipole,” an electrically short center-fed antenna containing a uniform current, $I$. The authors observed that the electric field of this device when used in transmitting was $E = \frac{j \omega \mu I}{4\pi} e^{-j\beta r} \Delta z \sin \theta \hat{\theta}$, where $\Delta z$ is the antenna length and $\beta$ is the free-space wavenumber. They continued by saying that “since $h$ contains information on the size of the antenna and the angular dependence of the radiation pattern we can write $E = \frac{j \omega \mu I}{4\pi} e^{-j\beta r} h$ where $h = \Delta z \sin \theta \hat{\theta}$.

The preceding is certainly not a proof that the last equation is the vector effective length of an ideal dipole. Indeed, had the writers used $E = \frac{j \omega \mu I}{2\pi} e^{-j\beta r} (\Delta z/2) \sin \theta \hat{\theta}$, which is certainly valid, they might have argued that $h = \frac{\Delta z}{2} \sin \theta \hat{\theta}$. The example gave no
clue as to how to obtain the effective length of an antenna of a different length, or one with a nonuniform current distribution.

This matter is treated further, on page 472, where effective length is redefined by a new equation, $V_A = -h^r_A \cdot E^j$ “where $h^r_A$ is the antenna effective length upon receiving.” We are warned that this differs from the previous definition, and told that the present one can be found in a doctoral dissertation, perhaps leading the reader to think that its derivation is difficult.

Moving to reference [2], we find that vector effective length (here called “[vector] effective height” $h_e$) is treated as a footnote on page 31, while the main material on pages 30 and 31 describes a scalar “effective height,” $h$. The defining equation is $V = hE$, which does not warn the reader that the voltage induced in the gap of the receiving antenna is a function of both the direction from which the incoming radiation is incident as well as the polarization of that radiation, although there is a remark about the antenna being “oriented for maximum response.” Having introduced this definition, the book asserts that because a half-wave dipole exhibits a sinusoidal current, its effective length is $2/\pi$ multiplied by its actual length. The logic of this statement is unclear. The matter of polarization mismatch is treated in the footnote.

On page 31, the book observes that $h$ now redefined as $h_e$, can be obtained from $I(z)$, the current along the antenna when used for transmitting. The equation given is
\[ h_e = \frac{1}{I_0} \int_0^{h_p} I(z) \, dz, \]

where \( h_p \) is the actual physical length and \( I_0 \) is the current now supplied in the gap by a generator. As can be seen from the derivation of Equation (20), the above equation is misleading, as it is valid only for one particular angle of incidence of the received wave. The same text also observes that “effective height is a useful parameter for transmitting tower type antennas,” but the use of such antennas assumes a conducting Earth and there is no indication of how the preceding formula accounts for its presence.

Looking at the third text, [3, p. 88], we find that the vector effective length \([l_e]\) “is a far field quantity and is related to the far zone-field \( \mathbf{E}_a \) radiated by the antenna, with current \( I_{in} \) in its terminals by...

\[ \mathbf{E}_a = -j \eta \frac{k I_{in}}{4\pi r} l_e e^{-jkr}. \]

The author maintains that the open-circuit voltage of a receiving antenna is given by \( V_{oc} = \mathbf{E}^i \cdot l_e \), where \( \mathbf{E}^i \) is the incident electric field. No attempt is made to prove that the \( l_e \) appearing in his first equation (which describes transmitting) can be used in the second (which describes receiving)—although he lists six references to literature that presumably give the connection.

Incidentally, had the author chosen to write \( \mathbf{E}_a = j \eta \frac{k I_{in}}{4\pi r} l_e e^{-jkr} \), his definition would have been in conformance with the one given here (see Equation (18)), as well as with common usage.
CONCLUSION

The preceding material is suitable for an introductory course in antennas. Because the radiated field of an electrically small loop antenna is essential to the discussion, it will perhaps be necessary to derive this field earlier in the course than is ordinarily done, in order to apply the method given here. As the field of a small loop is invariably taught in an introductory course in antennas, this juggling in the order of topics should create no hardship. The exposition of effective length presented leads seamlessly to the subject of effective area, and serves as a warning that one must be concerned with the possibility of polarization mismatch in applying \( A_{eff}(\theta, \phi) \).

method that we have introduced implicitly requires that the antenna the effective length of which we are seeking is one for which the transmitting pattern is known. This, in turn, requires that we know the current on the antenna when it is transmitting. In the case of wire or thin tubular antennas, especially dipoles, a considerable literature exists on this subject, e.g. [8].

In a more-advanced course, students can be encouraged to extend the subject of receiving antennas to more complicated situations in which the antenna is receiving an elliptically polarized wave, and/or when the antenna itself is capable of generating such a wave when transmitting. The problem of a
receiving wire antenna parallel to a reflecting surface might also be studied.

**APPENDIX: USE OF A GROUND PLANE**

A receiving antenna often consists of a vertical wire placed at right angles to a perfectly conducting ground plane, with a load placed between the bottom of the antenna and the plane. Suppose a plane wave is incident on this monopole arrangement, as shown in Figure 7a. We will assume that the electric field is polarized only along $\mathbf{a}_\theta$, as any field component along $\mathbf{a}_\phi$ would have no effect at the load. Proceeding as before, we will assume that this electric field is created by a loop antenna above the ground plane. The current on the loop antenna is given by Equation (3), and the incident field striking the monopole at its base is $E_0' \mathbf{a}_\theta$. This is not the total field experienced by the monopole at $z = 0$, as it fails to account for the electric field of the wave reflected off the ground plane. The total field at $z = 0$ is, of course, normal to the plane.

![Figure 7a. A receiving monopole above an infinite ground.](image)
To compute the voltage appearing at the gap between the bottom surface of the monopole and the ground plane, we apply the method of images. This means we eliminate the ground plane, and introduce the image of the monopole and that of the loop antenna just described. The result is shown in Figure 7b. The monopole plus its new image now comprise a dipole, and the voltage across the gap in this dipole is the sum of the voltages created by the original loop and its image. the voltage created across the gap by the upper loop is simply $E_0^i l_{\text{eff}}(\theta)$, where $l_{\text{eff}}(\theta)$ is the effective length of a dipole the upper half of which is the given monopole. By symmetry, the lower loop creates an identical voltage in the gap. The total voltage in the gap of the dipole is thus $2E_0^i l_{\text{eff}}(\theta)$. We return to Figure 7a and see that from a symmetry argument, the voltage existing at the base of
the monopole with respect to the ground plane is one-half the preceding result. The incident field, $E_0^i \mathbf{a}_\theta$, creates an open-circuit voltage $E_0^i l_{\text{eff}}(\theta)$, or $E_0^i \mathbf{a}_\theta \cdot l_{\text{eff}}$, where $l_{\text{eff}}(\theta)$ is the vector effective length of the corresponding dipole in the absence of the ground plane. To summarize:

*The vector effective length of a base-driven monopole antenna that is perpendicular to a perfectly conducting infinite ground plane is identical to the vector effective length of a dipole (in the absence of the ground plane) the upper half of which is that monopole.*

To take a simple example, we have shown above that a half-wave conventional dipole antenna has a vector effective length $l_{\text{eff}}(\theta) = \frac{\lambda \cos[(\pi/2) \cos \theta]}{\sin \theta} \mathbf{a}_\theta$, which means that a quarter-wave monopole, loaded at its base, has the identical vector effective length.

The voltage, $V_{lg}$, appearing in the gap between the lowest point on the monopole and the ground plane is

$$V_{lg} = \frac{\lambda \cos[(\pi/2) \cos \theta]}{\pi \sin \theta} \mathbf{a}_\theta \cdot (E_0^i \mathbf{a}_e)$$

$$= \frac{\lambda \cos[(\pi/2) \cos \theta]}{\pi \sin \theta} E_0^i ,$$

where one must keep in mind that $E_0^i \mathbf{a}_e$ is the *incident field at the base, not the total field*. The total field is in fact
perpendicular to the ground plane, and has the value
\[ E = -2E_0^\parallel \sin \theta a_z \]

at the base, as can be seen if we analyze the reflection of a plane wave from an infinite ground plane.

The effective area of a lossless antenna in a lossless environment is found from Equation (23) to be
\[ A_{\text{eff}}(\theta, \phi) = 30\pi \frac{|l_{\text{eff}}(\theta, \phi)|^2}{R_r}, \]

since \( R_\Omega = 0 \). Now a lossless monopole antenna driven above an infinite ideal ground plane has an effective length equal to that of the corresponding dipole, as noted above. However, it is well known that the radiation resistance, \( R_r \), of this monopole is one-half that of the corresponding dipole in the absence of the plane, since radiation takes place from the monopole only in the half space above the ground plane. We see from the preceding formula for \( A_{\text{eff}}(\theta, \phi) \), that the effective area of a monopole antenna driven over an ideal ground plane is twice that of the corresponding dipole in the absence of the ground plane. Since antenna gain is proportional to effective area, a monopole above an ideal ground plane has twice the gain of the corresponding dipole in the absence of the plane.

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